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Varagur S. V. Rajan [a] & Jules J. C. Picot [a]
[a] Department of Chemical Engineering, University of New Brunswick, Fredericton, N.B., Canada
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Non-Fourier Heat Conduction in a Nematic Mesophase†

VARAGUR S. V. RAJAN and JULES J. C. PICOT

Department of Chemical Engineering
University of New Brunswick
Fredericton, N.B., Canada

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Abstract—A brief discussion of non-linear (non-Fourier) heat conduction is given. Recent experimental data on nematic *p*-anisylidene *p*-aminophenylacetate are also presented to confirm the earlier reported unusual effects of temperature gradient and boundaries on the thermal conductivity of nematic mesophases such as *p*-azoxyanisole.

1. Introduction

The simplest approach for the transfer of energy by conduction is to postulate that the heat flux vector $\mathbf{q}$ is a function only of the temperature T and temperature gradient ∇T,

$$\mathbf{q} = \mathbf{q}(T, \nabla T) \qquad (1)$$

$$\nabla \cdot \mathbf{q} = 0 \quad \text{(steady state conservation of energy)} \qquad (1a)$$

The relation between $\mathbf{q}$ and ∇T is given in a general form as follows:

$$\mathbf{q} = -k\,\nabla T \qquad (2)$$

where k is the apparent thermal conductivity of the medium.

$$k = k(T, |\nabla T|) \qquad (3)$$

$|\nabla T|$ is the magnitude of the temperature gradient vector. These have been shown to be valid for isotropic, homogeneous media. The well known linear version of Eq. (2) requires that k be independent of ∇T. This linear law is known as the Fourier Law of heat conduction. Analogous relations are available for diffusive momentum and mass transfer, known as Newton's and Fick's Law respectively. Extensive discussions on these are given by Flumerfelt[3] and Slattery.[12] Very little is known about the nature of the non-linear or non-Fourier heat conduction process. We have no way of knowing *a priori* the nature

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of the functional relation $k(T, |\nabla T|)$ for any substance. A simplified molecular model for the material under consideration would be a starting point.

For an anisotropic medium, the above equation may be extended and written as follows[9]:

$$\mathbf{q} = -\mathbf{k}(T, |\nabla T|) \cdot \nabla T \qquad (4)$$

where $\mathbf{k}$ is a second order thermal conductivity tensor. If the medium is heterogeneous, we have

$$\mathbf{k} = \mathbf{k}(T, \mathbf{r}, |\nabla T|) \qquad (5)$$

where $T = T(\mathbf{r})$ and $\mathbf{r}$ is the position vector.

In contrast with the numerous studies of non-Newtonian momentum transport (e.g. viscoelastic fluids etc.) very little experimental evidence is available to show that the thermal conductivity depends upon the temperature gradient. One possible reason for this (if some materials such as liquid crystals have such a dependence[8]) generally is the lack of a simple experiment by which $k(T)$ is distinguished from $k(|\nabla T|)$. Recently, Flumerfelt and Slattery[3] and Patharkar *et al.*[8] experimentally showed that k depends on $|\nabla T|$ for powdered solid Sauereisen cement No. 31 and nematic *p*-azoxyanisole respectively. There seems to be no other experimental evidence of this unusual phenomenon of non-linear heat conduction in liquids, though non-linear effects are common in rarefied gases.

Patharkar *et al.*[8] proposed that the effect of temperature gradient on heat conduction is perhaps due to a reorienting effect of the temperature gradient on the nematic liquid crystal. It has been stated in the literature[5,9,14] that the thermal conductivity tensor is related to the director (unit vector) $\mathbf{n}(\mathbf{r})$ of the nematic spheroidal swarms as follows:

$$\mathbf{k} = k_0(\mathbf{I} + \lambda_1 \mathbf{A}) \qquad (6)$$

where $\left.\begin{aligned}\mathbf{A} &= \text{anisotropy tensor}^{(4)} \\ &= \langle \mathbf{n}(\mathbf{r})\mathbf{n}(\mathbf{r})\rangle - \mathbf{I}/3\end{aligned}\right\} \qquad (6a)$

k_0 = isotropic or random oriented conductivity

λ_1 = material constant (conductivity anisotropy)[11]

$\mathbf{I}$ = unit tensor

$\langle\ \rangle$ = ensemble average for all orientations as described by a suitable distribution function $f(\mathbf{n}(\mathbf{r}))$. ($\mathbf{n}$ may also be time dependent.)

Transverse isotropy is assumed in the above—i.e., $\mathbf{n}$ is not distinguishable from $-\mathbf{n}$ (invariance to mirror reflections) for the long axis of the swarms. In view of the interfacial and temperature gradient effects,[8,9] $\mathbf{A}$ depends on $\mathbf{r}$, T and $|\nabla T|$. Experimental evidence of this reorientation by $|\nabla T|$ has been given by many workers.[1,5,13] These experiments were subject to criticism because of possible convective effects. These and other aspects are discussed adequately in the review article by Rajan and Picot.[11] As discussed in the preceding review, attempts have been made by many workers to explain this orientation by temperature gradient for cholesteric mesophases[6] and for nematics[2] with limited success. Leslie[7] suggested that the assumption of uncoupled mechanical (orientational) and thermal effects may be amended by incorporating an intrinsic body force (orientation) term as a non-linear function of $|\nabla T|$ in the constitutive equation. A similar suggestion was made by Patharkar *et al.*[8] for the equation of change for $\mathbf{A}$[14] as follows:

$$\frac{D\mathbf{A}}{Dt} = \lambda\nabla^2\mathbf{A} - \frac{\mathbf{A}}{\tau} + \mathbf{R}_A(|\nabla T|) \tag{7}$$

where D/Dt = convected derivative

λ = interfacial effect diffusion constant for orientation

τ = thermal reorientation or relaxation time constant

$\mathbf{R}_A$ = orientation generation term

In view of the weak dependence of the thermal conductivity on T for a narrow temperature range, the effect of T is not introduced in the above equation.[8,11] For a steady state non-flow system with large volume to surface ratio (i.e. interfacial effect can be neglected for large distances from the surface), we have

$$\mathbf{A} \simeq \tau R_A(|\nabla T|) \tag{8}$$

The functional relation for R_A $(|\nabla T|)$ is still under investigation.

2. Experimental Results

To investigate further this aspect of non-Fourier conduction and orientation phenomena in nematic liquid crystals, thermal conductivity measurements were made on nematic p-anisylidene p-aminophenylacetate (APAPA) in the cell described earlier.[8] The cell is of a non-flow, steady state, parallel plate type.

TABLE 1 Thermal Conductivity of APAPA (temperature gradient effect) [T_c in °C; q in cal/sq cm sec $(T_p - T_c)/L$ in °C/cm and k_e in cal/sec cm °C]

L	T_c	$q \times 10^4$	$(T_p - T_c)/L$	$k_e \times 10^4$	L	T_c	$q \times 10^4$	$(T_p - T_c)/L$	$k_e \times 10^4$
	78.78	2.997	0.888	3.38		79.76	36.91	10.14	3.64
	79.18	4.306	1.297	3.32		80.21	63.58	17.90	3.55
0.3175	79.07	4.807	1.463	3.29	0.0254	80.46	93.96	27.65	3.40
	79.04	8.005	2.441	3.28		79.77	119.3	34.85	3.42
	78.68	12.00	3.511	3.42		80.30	70.37	19.07	3.69
	80.13	12.69	3.771	3.37		80.97	132.7	38.14	3.48
0.1016	80.23	18.56	5.570	3.33	0.0127	81.44	193.8	56.35	3.44
	80.23	19.72	6.090	3.24					
	80.32	25.56	8.409	3.04					
	79.49	17.60	4.898	3.60		80.02	110.3	29.13	3.79
	79.75	26.33	7.503	3.51		80.53	202.5	55.03	3.68
0.0508	79.72	35.87	10.45	3.43	0.0064	80.41	312.9	86.69	3.61
	79.80	47.27	14.13	3.35		80.45	397.9	110.6	3.60
	79.52	56.50	17.40	3.25		80.43	459.6	129.7	3.54

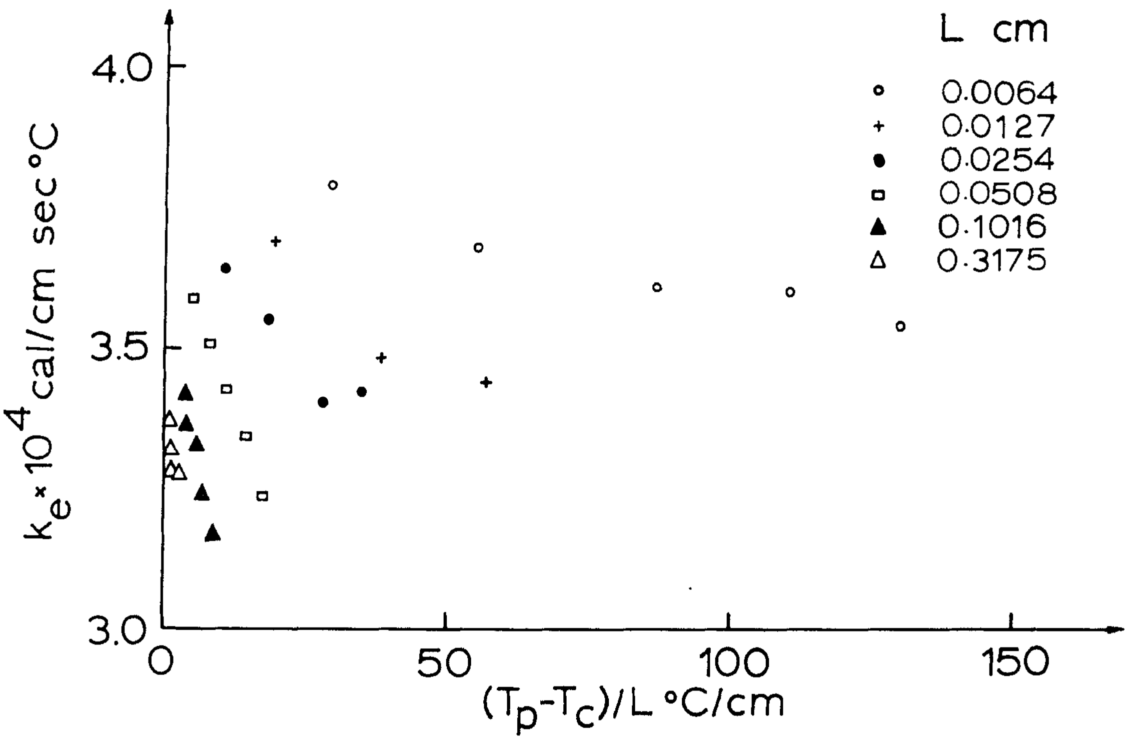

Figure 1. Temperature Gradient Effect on Thermal Conductivity of APAPA. T_p—Top surface temperature; T_c—bottom surface temperature; L—cell gap (sample thickness); q—heat flux and k_e—effective thermal conductivity.

Table 1 gives the data on thermal conductivity. A decrease in thermal conductivity with increase in cell gap and also with increase in temperature gradient at all cell gaps is observed. This is similar to the observations for PAA.[8] Figure 1 is a plot of this data. This affirms the temperature gradient effect on heat conduction (non-Fourier effect) in nematic liquid crystals.

The interfacial effect for very low heat fluxes (or $|\nabla T|$) was analyzed for this one-dimensional system with a "swarm-continuum" model by Rajan *et al.*[9,10] The coupling effect of the geometry of the system and $|\nabla T|$ is yet to be analyzed. These data show that the effect of $|\nabla T|$ decreases with decreasing thickness of the sample when the interfacial orientation is predominant. The increased effect of $|\nabla T|$ for very large gaps is not clear as one would expect the disorienting effect of thermal motion or rotational Brownian motion to be predominant. It must be noted that there is an upper limit on the $|\nabla T|$ for each gap to avoid natural convection effects.[11] Also, there are two additional reasons for limiting the applied $|\nabla T|$ given as follows:

(i) the entire sample must remain in the nematic range

and (ii) the effect of T should be kept small so that only $|\nabla T|$ effect may be observed.

For small gaps the above reasons, and for large gaps the convection effect, seem to be the deciding factors for limiting the applied heat flux.

3. Concluding Remarks

We have presented additional data to reinforce the earlier reported non-Fourier heat conduction effect. Theoretical explanation of this phenomenon for nematic mesophases in terms of an orientation effect is still at an infant stage. There are two requirements in this field at this stage of the art of liquid crystals:

(a) further experimental data on other nematics. We are investigating this for MBBA, and
(b) an independent optical or other means of showing the reorienting effect of heat conduction.

Such a study will be an important contribution to the field of applied mechanics of interest to both engineers and scientists.

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REFERENCES

1. Anzelius, A., Uppsala Univ. *Årsskr., Mat. OchNaturvet.* 1 (1931).
2. Davison, L. and Amos, D. E., *Phys. Rev.* **183,** 288 (1969).
3. Flumerfelt, R. W. and Slattery, J. C., *A.I.Ch.E.J.* **15,** 291 (1969).
4. Frenkel, J., " Kinetic Theory of Liquids ", Dover, New York (1955).
5. Lehmann, O., *Ann. Phys.* **2** (4), 649 (1900).
6. Leslie, F. M., *Proc. Roy. Soc.* **A307,** 359 (1968).
7. Leslie, F. M., reply to V. S. V. Rajan, " Continuum Theory of Thermotropic Systems " (General Discussion). Symposium on Liquid Crystals Faraday Society, London, December 13–14 (1971).
8. Patharkar, M. N. *et al., Mol. Cryst. and Liq. Cryst.* **15,** 225 (1971).
9. Rajan, V. S. V. and Picot, J. J. C., *Mol. Cryst. and Liq. Cryst.* **17,** 109 (1972).
10. Rajan, V. S. V. *et al., Mol. Cryst. and Liq. Cryst.* **18,** 279 (1972).
11. Rajan, V. S. V. and Picot, J. J. C., " Thermal Transport Phenomena in Nematic Liquid Crystals ", to be published in *Mol. Cryst. and Liq. Cryst.*, in press.
12. Slattery, J. C., *Appl. Sci. Res.* **A12,** 51 (1963); **A14,** 420 (1965).
13. Stewart, G. W. *et al., Phys. Rev.* **58,** 174 (1960).
14. Sullivan, J. T., Ph.D. Thesis, University of Minnesota (1966).